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FINAL STATE INTERACTIONS OF $B \rightarrow DK$ DECAYS**Hanqing Zheng**P. Scherrer Institute, 5232 Villigen, Switzerland¹**Abstract**

We study the final state strong interactions of the $B \rightarrow DK$ decay processes, using the Regge model. We conclude that the final state interaction phases are very small, typically a few degrees. Neglecting final state interactions in obtaining the weak decay amplitudes is a good approximation.

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The importance of studying the final state strong interactions of the two body non-leptonic B decays is due to the desire of determining the relevant CKM matrix elements and of studying the CP violation effects in the B decay processes. In the first case the final state strong interactions will modify the value of the CKM matrix elements extracted from the bare weak decay amplitudes. In the latter case, in order to have observable CP violation effects, it is necessary to find the difference between the decay rate of a process $i \rightarrow f$ and the rate of its charge conjugate process, $\bar{i} \rightarrow \bar{f}$ where there have to be at least two interfering amplitudes with different weak interaction phases as well as different strong interactions phases. By writing the bare weak interaction T matrix as,

$$A = g_1 A_1 + g_2 A_2 , \quad (1)$$

where the g_i are the weak couplings, we have, after taking the final state strong interactions into account,

$$\langle f | A | i \rangle = g_1 A_1 e^{i\alpha_1} + g_2 A_2 e^{i\alpha_2} , \quad (2)$$

$$\langle \bar{f} | A | \bar{i} \rangle = g_1^* A_1 e^{i\alpha_1} + g_2^* A_2 e^{i\alpha_2} , \quad (3)$$

where the α_i are the strong interaction phases. The difference in the decay widths of $i \rightarrow f$ and $\bar{i} \rightarrow \bar{f}$ is,

$$\Gamma - \bar{\Gamma} \sim \text{Im}(g_1^* g_2) \sin(\alpha_1 - \alpha_2) . \quad (4)$$

We see that whether there are observable CP violating effects depends crucially on the strong interaction phase difference between the two interfering amplitudes.

Despite of the importance of the final state strong interactions in B decay processes, it is however difficult to deal with and poorly understood. In the K decay system the final state strong interaction phase difference can be estimated reliably using the low energy effective theory of strong interactions. For the D decay system in which the center of mass energy is too large to apply the low energy effective theory, a single resonance dominance assumption was used in studying the final state strong interactions. Under this assumption, the strong interaction amplitude is simply parametrized as the function of the decay rate of the resonance and the couplings between the resonance and the interacting particles. The phase difference can therefore be estimated in a reasonable range of parameter space [1]. This method can not be successful for the B decays because we know from QCD that the number density of resonances is an increasing function of the mass.

In this paper, we study the final state strong interactions of the two body non-leptonic B decay processes using the Regge model analysis ². This method is based

²A brief discussion based on the Regge model was given in ref. [2]

upon the strong interaction duality argument: When the center of mass energy gets high enough, the summation of the contributions from the s-channel resonances to the amplitude is equivalent to the summation of the contributions from leading t-channel Reggeon exchanges. The Regge model has been proven to be very successful in explaining high energy hadron scattering in the small $-|t|$ region [3]. As an example, in the case of πN elastic scattering resonance models only work when the center of mass energy is less than $s \simeq 6\text{GeV}^2$ and fail badly when s exceeds 10GeV^2 where the Regge model starts to work very well [4] (In charge exchange processes it works even at lower energies). In order to have a clear insight on the physics concerned, we limit ourselves to the processes without penguin diagram contributions. The absorbtion part of the amplitudes in such a case come only from "soft" final state strong interactions. Especially, we discuss the $B \rightarrow D\bar{K}$ and $B \rightarrow \bar{D}\bar{K}$ processes although applications of our method to any two body decay process are straightforward.

The relation between the full physical amplitude when the final state interactions are taken into account and the bare amplitude is given by the final state theorem of Migdal and Watson [5],

$$A_{tot} = S^{\frac{1}{2}} A_{bare} . \quad (5)$$

where S is the final state strong interaction S matrix for the given partial wave ($J = 0$ in the present case).

The final states are classified according to their flavor quantum numbers. For a fixed final state f various intermediate states can contribute coherently to the rescattering. For example for the final state D^+K^- both D^+K^- and $D^0\bar{K}^0$ are possible as intermediate states leading to a 2×2 S matrix. While for \bar{D}^0K^- , $D^-\bar{K}^0$ case things are more complicated since the $D_s^-\eta$ and $D_s^-\pi^0$ also contribute and the S matrix is 4×4 ³. The Pomeron and Reggeon contributions to the s channel scattering amplitude $A(s,t)$ can be written down systematically using the crossing symmetry between the s channel and the t channel amplitudes, the line-reversal law [3], and the $SU(3)$ relations [6],

$$A(\bar{D}^0K^- \rightarrow \bar{D}^0K^-) = P + \rho + A_2 + \omega + f , \quad (6)$$

$$A(\bar{D}^0\bar{K}^0 \rightarrow \bar{D}^0\bar{K}^0) = P - \rho - A_2 + \omega + f , \quad (7)$$

$$A(D^-\bar{K}^0 \rightarrow D^-\bar{K}^0) = P + \rho + A_2 + \omega + f , \quad (8)$$

$$A(D^+\bar{K}^0 \rightarrow D^+\bar{K}^0) = P - \rho + A_2 - \omega + f , \quad (9)$$

$$A(D^+K^- \rightarrow D^+K^-) = P + \rho - A_2 - \omega + f , \quad (10)$$

³There are many other states with the same total spin and flavor quantum numbers, including two body states with excited particles and multi-particle states. As an approximation we systematically neglect them. We will discuss their influence later in this paper.

$$A(D^0 \bar{K}^0 \rightarrow D^0 \bar{K}^0) = P + \rho - A_2 - \omega + f , \quad (11)$$

$$A(D^0 K^- \rightarrow D^0 K^-) = P - \rho + A_2 - \omega + f , \quad (12)$$

$$A(D_s^- \pi^0 \rightarrow D_s^- \pi^0) = P , \quad (13)$$

$$A(D_s^- \pi^+ \rightarrow D_s^- \pi^+) = P , \quad (14)$$

$$A(D_s^- \eta \rightarrow D_s^- \eta) = P + \frac{8}{3}f' , \quad (15)$$

where the P , ρ , ω , A_2 , f and f' denote the contribution from the Pomeron, ρ , ω , A_2 , f and f' Reggeon exchanges to the the amplitude, respectively. For the charge exchange processes, we have

$$A(\bar{D}^0 K^- \rightarrow D^- \bar{K}^0) = 2\rho + 2A_2 , \quad (16)$$

$$A(D^+ K^- \rightarrow D^0 \bar{K}^0) = -2\rho + 2A_2 , \quad (17)$$

$$A(D_s^- \eta \rightarrow \bar{D}^0 K^-) = -\sqrt{6}K^* - \sqrt{\frac{2}{3}}K^{**} , \quad (18)$$

$$A(D_s^- \eta \rightarrow D^- \bar{K}^0) = -\sqrt{6}K^* - \sqrt{\frac{2}{3}}K^{**} , \quad (19)$$

$$A(D_s^- \pi^0 \rightarrow D^- \bar{K}^0) = \sqrt{2}(K^* - K^{**}) , \quad (20)$$

$$A(D_s^- \pi^0 \rightarrow \bar{D}^0 K^-) = \sqrt{2}(K^{**} - K^*) , \quad (21)$$

$$A(D_s^- \pi^+ \rightarrow \bar{D}_0 \bar{K}_0) = 2(K^{**} - K^*) , \quad (22)$$

where the K^* and K^{**} denote the K^* and K^{**} Reggeon contributions, respectively. The $SU(3)$ coefficients are written explicitly in above formulas. Please notice that the Pomeron couplings may depend on the different flavour content while various Reggeon contributions are related to each other by the strong exchange degeneracy (SED) requirement [3]. Especially, the magnitude of each Reggeon contribution in an amplitude should be equal. The SED is known to hold quite well for vector and tensor Reggeon exchange processes especially when the strange quark is involved. Sign differences between Reggeon contributions in the above formulas lead to the cancellation between the imaginary part of the Reggeon contributions to s channel exotic amplitudes. This is just a manifestation of the fact that the absence of an imaginary part in A^R implies that there is no s channel resonance, because of the duality argument.

We parametrize the Pomeron and the Reggeon exchange amplitudes in the small $-|t|$ region in the following way:

$$P = \beta^P(t) \left(\frac{s}{s_0} \right)^{\alpha^P(t)} e^{-i\frac{\pi}{2}\alpha^P(t)} , \quad (23)$$

$$R = \beta^R(t) \left(\frac{s}{s_0} \right)^{\alpha^R(t)} \frac{\pm 1 - e^{-i\pi\alpha^R(t)}}{\sin \pi\alpha_0^R} , \quad (24)$$

$$\beta^P(t) = \beta^P e^{a^P t} \quad \text{and} \quad \beta^R(t) = \beta^R e^{a^R t} , \quad (25)$$

where \pm sign refers to odd/even signatures of the exchanging Reggeons. The $SU(3)$ invariant couplings β^R are normalized such that it is just the value of the residue function of the Reggeon-matter coupling at origin, i.e., $t = 0$. In accordance with the traditional treatment we use an exponential form of parametrization to characterize the form factor of the Reggeon-matter couplings. The parameter a^R can be estimated using the Veneziano model [7] for meson-meson scatterings,

$$a^R = -\frac{\Gamma'(1 - \alpha_R^0)}{\Gamma(1 - \alpha_R^0)} \alpha_R' \quad (26)$$

which obeys approximately the relation: $a^R = \frac{1}{m_R^2}$ where m_R is the mass of the corresponding elementary particle of the Reggeon R. We will approximate a^R by $1/m_R^2$ in the following.

It is worth pointing out that our parametrization, eq. (24), is different from what is usually used in the literature:

$$A^R(s, t) = -x\beta^R e^{a^R t} \left(\frac{s}{s_0}\right)^{\alpha^R(t)} e^{-i\pi\alpha^R(t)/2} \quad (27)$$

with $x = 1/-i$ for even/odd signatures, respectively. The reason is that as $\alpha_R^0 \neq 0.5$, the parametrization eq.(27) violates SED by introducing an imaginary part to the Regge amplitude for a s-channel exotic process and is therefore not adequate for the present discussion. For the trajectory functions $\alpha^P(t)$ and $\alpha^R(t)$ we use the following values [3, 8],

$$\alpha^P(t) \simeq 1.08 + 0.25t, \quad (28)$$

$$\alpha^\rho(t) \simeq 0.44 + 0.93t, \quad (29)$$

$$\alpha^{K^*}(t) \simeq 0.3 + 0.93t, \quad (30)$$

$$\alpha^{f'}(t) = 0.1 + 0.93t. \quad (31)$$

To estimate the residue functions $\beta^P(t)$ and $\beta^R(t)$ we assume factorization. That is, for a process $AB \rightarrow CD$ we have,

$$\beta_{AB \rightarrow CD}(t) = \beta_{AC}(t)\beta_{BD}(t), \quad (32)$$

both for $\beta^P(t)$ and $\beta^R(t)$. In the literature, the Pomeron coupling to the pion is taken as a single pole form in the small $-|t|$ region [9],

$$\beta_\pi^P(t) = \beta_\pi^P \frac{1}{1 - t/0.71}. \quad (33)$$

The t dependence of Pomeron couplings to D and K mesons are not known experimentally, we simply assume they are the same as that in eq.(33)⁴. For the residue

⁴Roughly speaking, the formfactor is the Fourier transformed version of the size of the hadron. Since the π , D and K mesons both involve light quarks their size is expected to be similar.

function of the Pomeron amplitude $\beta^P(t)$, we take

$$\beta^P(t) = \beta_D^P \beta_K^P \simeq \beta_\pi^P \beta_\pi^P \sim \left(\frac{1}{1 - t/0.71} \right)^2 \simeq e^{a^P t} = e^{2.82t} . \quad (34)$$

The value of β_{DK}^P can be estimated as follows. From πp and Kp data one gets,

$$\beta^P(su) \simeq \frac{2}{3} \beta^P(uu) , \quad (35)$$

and

$$\beta^P(cu) \simeq \frac{1}{10} \beta^P(uu) , \quad (36)$$

from [8]. Using isospin invariance

$$\beta^P(uu) = \beta^P(ud) = \beta^P(dd) , \quad (37)$$

and the additive quark counting rule we obtain

$$\beta^P(DK) = \beta^P(cu) + \beta^P(cs) + \beta^P(us) + \beta^P(uu) \simeq 12.0 . \quad (38)$$

The coupling constant β^R in eq. (25) cannot be estimated from $SU(3)$ constraints, since the D meson is a $SU(3)$ triplet and may have different coupling comparing with the $SU(3)$ octet mesons. It can however be fixed from the $SU(4)$ relation

$$\beta_D^R = \beta_K^R , \quad (39)$$

provided that the coupling constant β_K^R can be estimated from NN , πN and KN scattering data. Eq.(39) is also a direct consequence of the vector meson coupling universality, which is respected satisfactorily in the $\pi(K)N$ processes.

The Regge pole amplitudes for $\pi(K)N$ elastic scattering can be written down analogously. We have fitted the experimental data on the $\pi(K)N$ total cross-sections and find that $\beta^R \simeq 3.11$.

At this stage we are able to calculate the two body scattering Regge amplitudes given above which is however not directly applicable for studying the final state interactions of the B decay system. Because the B meson has spin 0, the corresponding scattering amplitude should be the s-partial wave projection of the full Regge amplitude:

$$A_{J=0}(s) = \frac{s}{16\pi\lambda} \int_{-t_-}^0 A(s, t) dt , \quad (40)$$

where

$$\lambda \equiv \lambda(s, m_1^2, m_2^2) = s^2 + m_1^4 + m_2^4 - 2sm_1^2 - 2sm_2^2 - 2m_1^2 m_2^2 , \quad (41)$$

and

$$t_- = \frac{\lambda}{s} . \quad (42)$$

Before performing the numerical calculation we first remark on the reliability of our method. It is known that the Regge parameterization is only valid in the small $-|t|$ region. In order to obtain the s-wave amplitude we need to perform the integration over the whole $-|t|$ region, and further, low partial wave projections of the Regge amplitudes are considered as the most unreliable part of the theory. However, the large $-|t|$ contributions to the total scattering amplitude is suppressed by $1/s$, therefore the uncertainty due to the invalidity of the Regge amplitude at large $-|t|$ region and the dependence of our results on the parameterization form in the small $-|t|$ region should not be important. For the large $-|t|$ region Regge cuts and exchange channel Reggeons dominate. The Regge trajectory in the u-channel are D^* and $D^{**}(2460)$ in our case. However, these u-channel Reggeons have a very small intercept α_R^0 :

$$\alpha_{D^*}^0 = \alpha_{D^{**}}^0 = 1 - \frac{m_{D^*}^2}{m_{D^{**}}^2 - m_{D^*}^2} \simeq -1, \quad (43)$$

their contributions are therefore negligible.

For the Regge cuts, it was realized long ago that one of the main defects of Regge poles is that they give too large a contribution to the low partial waves. Some extra absorption provided by the Regge cuts is necessary. However, the absorption effects in our case (meson-meson scattering) should not be large either. There is strong experimental evidence indicating that the absorption effects are much smaller in πN scattering than in NN scattering. It is natural to expect that in meson meson scattering these absorption effects are even smaller. This may be understood from the simple Reggeized absorption model [10] despite of the fact that Regge cuts are less well known theoretically than Regge poles. In the simple Reggeized absorption model the cuts generated by two Pomerons or one Pomeron one Reggeon exchange contribute to the full s-partial wave amplitude as:

$$A^{P+P} \otimes P = A^P \left(1 - \frac{\lambda_P \beta^P}{16\pi c_P s_0} \left(\frac{s}{s_0} \right)^\epsilon \right), \quad (44)$$

and

$$A^{R+R} \otimes P = A^R \left(1 - \frac{\lambda_R \beta^P}{8\pi c_P s_0} \left(\frac{s}{s_0} \right)^\epsilon e^{-i\frac{\pi}{2}\epsilon} \right), \quad (45)$$

where $\epsilon = 0.08$, $c_P = a^P + \alpha'_P \left(\log(\frac{s}{s_0}) - \pi/2i \right)$. The phenomenological enhancement factor λ characterizes the contribution from quasi-elastic intermediate states. We read off from eqs.(44), (45) that the contribution from the cuts is proportional to β^P which is much smaller in our case, $\beta_{DK}^P \simeq 12$, than in the case of πN scattering. This is crucial to reduce significantly the correction to the s wave amplitude, different from the πN scattering case. For example, taking $\lambda_R = 2$ in the latter case will lead

to a complete absorption in the s-wave amplitude, $A_s \sim 0$ while the same value of the λ_R parameter only reduces $A_s(DK)$ by $\sim 30\%$. Even though we have poor knowledge in estimating the λ_R parameter we expect this amount of reduction is resonable, as an upper limit. The λ_P parameter may be safely neglected. According to ref. [9] the inclusion of the $P \otimes P$ cut in the NN scattering case only leads to a small change of the β parameter.

Because of the above arguments we use in the following only the simple Regge pole model to evaluate the effects of the final state interactions of the B decay processes from the final state theorem, eq.(5). In the practical calculation it is convenient to study the problem in the strong interaction eigenstates for which the S matrix has a diagonal form. For the $D\bar{K}$ case we have

$$A(D\bar{K}) = \begin{pmatrix} A_{I=0}(D\bar{K}) & 0 \\ 0 & A_{I=1}(D\bar{K}) \end{pmatrix}, \quad (46)$$

where $A_{I=0} = P + 2(\rho - A_2)$, $A_{I=1} = P + 2(A_2 - \rho)$ and $|D\bar{K} >_{I=0} = \frac{1}{\sqrt{2}}(|D^+ K^- > - |D^0 \bar{K}^0 >, |D\bar{K} >_{I=1} = \frac{1}{\sqrt{2}}(|D^+ K^- > + |D^0 \bar{K}^0 >.$ For the $\bar{D}\bar{K}$ case, there are two degenerate states of each isospin which are,

$$|0^1 > = -\frac{1}{\sqrt{2}}(|\bar{D}^0 K^- > + |D^- \bar{K}^0 >), \quad |0^2 > = |D_s^- \eta >, \quad (47)$$

$$|1^1 > = -\frac{1}{\sqrt{2}}(|\bar{D}^0 K^- > - |D^- \bar{K}^0 >), \quad |1^2 > = |D_s^- \pi^0 >. \quad (48)$$

The $J = 0$ S matrices, $S_{ij} = \delta_{ij} + 2i\sqrt{\rho_i \rho_j} A_{ij}^{J=0}$ ($\rho_i = \sqrt{\lambda_i}/s$) are the following:

$$\begin{pmatrix} \langle 0^1 | 0^1 > & \langle 0^1 | 0^2 > \\ \langle 0^2 | 0^1 > & \langle 0^2 | 0^2 > \end{pmatrix}, \quad \begin{pmatrix} \langle 1^1 | 1^1 > & \langle 1^1 | 1^2 > \\ \langle 1^2 | 1^1 > & \langle 1^2 | 1^2 > \end{pmatrix}. \quad (49)$$

In the strong interaction eigenstates S is parametrized as,

$$S^{J=0} \equiv \text{diag}(\eta_I e^{2i\delta_I}), \quad (50)$$

The η_I parameter characterizes the inelasticity of the given process.

The numerical results for the parameters given above are⁵,

$$\eta_0(\bar{D}\bar{K}) = 0.81, \quad \delta_0 = -3.3^\circ; \quad \eta_1(\bar{D}\bar{K}) = 0.84, \quad \delta_1 = 1.8^\circ; \quad (51)$$

⁵The S matrix is equal to,

$$\begin{pmatrix} a & b \\ b & a' \end{pmatrix},$$

with $a - a', b \sim O(10^{-1})a$. The $S^{1/2}$ matrix is of the following form:

$$\begin{pmatrix} x & y \\ y & x' \end{pmatrix},$$

with $x, x' \gg y$. Approximately $x = \sqrt{a}$, $y = b/2\sqrt{a}$ and $x' = \sqrt{a} + (a' - a)/2\sqrt{a}$. The numerical

$$\eta_0(D_s^- \eta) = 0.85, \delta_0 = -1.7^\circ; \eta_1(D_s^- \pi^0) = 0.83, \delta_1 = -1.5^\circ; \quad (52)$$

$$\eta_0(D\bar{K}) = 0.83, \delta_0 = 2.2^\circ; \eta_1(D\bar{K}) = 0.83, \delta_1 = -1.8^\circ. \quad (53)$$

From above we see that the phase shifts δ are very small (modulo $n\pi$, of course). This can be understood qualitatively: At the center of mass energy m_B the Pomeron contribution is dominant which gives almost a purely imaginary contribution to the A amplitude and therefore, the S matrix elements are almost purely real. Furthermore, in our case the Pomeron contribution is much smaller than in the πN or NN case because of the smaller couplings. Only small cancellation occur in the S matrix elements between the Pomeron contribution (mainly a negative real value) and unity (the value of the S matrix element in the limit of vanishing interaction).

We notice that in our results the η parameters are slightly less than 1, which indicates that we have neglected some final states in addition to those considered. In order to restore unitarity one has to take them into account which will lead to corrections to the relations between the bare amplitudes and the full amplitudes obtained above. The calculation of the effects brought by these states go beyond the ability of the present model analysis. In the following we try to give a simple qualitative estimate.

Consider a given strong interaction eigenstate $|a\rangle$ which diagonalizes the S matrix in the incomplete basis like the states that we have considered: $\langle a|S|a\rangle = \eta e^{2i\delta_1}$ and $\eta < 1$. Since η is less than 1 unitarity tells us that in the complete basis the S matrix is still nondiagonal. Many new states can contribute to restore unitarity by introducing the non-diagonal matrix element $\langle new|S|a\rangle$. Now we assume that all the effects of these states are characterized by a single effective state $|a'\rangle$ which, together with $|a\rangle$ leads to a 2×2 unitary S matrix which can be simply parametrized as,

$$\begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}. \quad (54)$$

The solution of $S^{1/2}$ is

$$\begin{pmatrix} x e^{i\alpha_1} & i\sqrt{1-x^2}e^{i\frac{(\delta_1+\delta_2)}{2}} \\ i\sqrt{1-x^2}e^{i\frac{(\delta_1+\delta_2)}{2}} & x e^{i\alpha_2} \end{pmatrix}. \quad (55)$$

results are,

$$S_{I=0}^{1/2}(\bar{D}\bar{K}) \simeq \begin{pmatrix} 0.87 - 3.58i \cdot 10^{-2} & 1.11 \cdot 10^{-2} - 4.19i \cdot 10^{-2} \\ 1.11 \cdot 10^{-2} - 4.19i \cdot 10^{-2} & 0.95 - 4.40i \cdot 10^{-2} \end{pmatrix},$$

$$S_{I=1}^{1/2}(\bar{D}\bar{K}) \simeq \begin{pmatrix} 0.91 + 3.06i \cdot 10^{-3} & 2.76i \cdot 10^{-2} \\ 2.76i \cdot 10^{-2} & 0.93 + 2.52i \cdot 10^{-3} \end{pmatrix}.$$

We see that the nondiagonal elements are much smaller than the diagonal one.

where

$$x = \sqrt{\frac{1 + 2\eta \cos(\delta_1 - \delta_2) + \eta^2}{2(1 + \eta \cos(\delta_1 - \delta_2))}} \quad (56)$$

and

$$\alpha_{1,2} = \frac{\delta_1 + \delta_2}{2} \pm \sin^{-1} \left(\frac{\eta \sin(\delta_1 - \delta_2)}{\sqrt{1 + 2\eta \cos(\delta_1 - \delta_2) + \eta^2}} \right) . \quad (57)$$

The full physical amplitude we are interested in $\langle a|B \rangle^f$ can then be written as,

$$\langle a|B \rangle^f = x e^{i\alpha} \langle a|B \rangle^b + \sqrt{1 - x^2} e^{i(\frac{\delta_1 + \delta_2}{2})} \langle a'|B \rangle^b . \quad (58)$$

From eqs. (56) and (57) we see that x is very close to 1 and α_1 very close to δ_1 when $\eta \sim 0.8$ within a reasonable range of the δ_2 parameter, say, $|\delta_2| < 30^\circ$ (remember that δ_2 should also not be very large because of the Pomeron dominance). Therefore the phase shift α_1 remains small. This is in agreement with the present experimental evidence. Furthermore since $\langle a'|B \rangle$ represents an averaged value of contributions from many states it is expected that large cancelation between different amplitudes should occur which leads to $|\langle a'|B \rangle / \langle a|B \rangle| \ll 1$. Therefore we can conclude from eq. (58) that the bare weak decay amplitude is a good approximation to the full physical amplitude, accurate up to, roughly speaking, about 10 percent.

To conclude, the partial wave elastic unitarity is a good approximation for the DK scatterings at $s = M_B^2$ even though we know that "Pomeron dominance" implies inelasticity. In our special case it happens that these two statements are consistent. One simple way to understand qualitatively the smallness of $\Delta\delta_I (= \delta_1 - \delta_0)$ is that only the Reggeon contribution is isospin dependent. Therefore it is proportional to $\Delta R_I / (1 - P)$ and be a small quantity. The above analysis can be applied to other processes. In $D\pi$ processes similar qualitative results should hold. In the $\pi\pi$ and $K\pi$ cases the Pomeron contribution gets larger (by a factor of ~ 2), the value of the η parameter is further decreased ($\eta \sim 0.6 - 0.7$). Uncertainties raised from inelasticity are more serious and sound conclusions are difficult to make.

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